**Name and Surname** : ...........................................................................................

**Grade/Class** : 12/......... **Mathematics Teacher** : ..............................

Hudson Park High School

GRADE 12

# MATHEMATICS

June Paper 1

Marks : ____ / 150

Time : 3 hours

Examiner : SLT

Date : June 2018

Moderator(s) : FRD and PHL

## INSTRUCTIONS

1. Illegible work, in the opinion of the marker, will earn zero marks.
2. Number your answers clearly and accurately, exactly as they appear on the question paper.
3. **NB**
   - **Leave *2 lines* open between each of your answers.**
   - **Start each new QUESTION at the *top* of a new page.**
4. **NB**
   - **Fill in the details requested on the front of this Question Paper.**
   - **Detach the Answer Sheet for Question 7 and fill in the details requested on it.**
   - ***Do not staple your Question Paper and Answer Pages together.***
5. Employ relevant formulae and show all working out. Answers alone may not be awarded full marks.
6. (Non-programmable and non-graphical) Calculators may be used, unless their usage is specifically prohibited.
7. Round off answers to 2 decimal places, where necessary, unless instructed otherwise.
8. If (Euclidean) Geometric statements are made, reasons must be stated appropriately.

## QUESTION 1

1.1. Solve for $x$ :

1.1.1. $x^2 = 3x$ ( 2)

1.1.2. $2x = \dfrac{7}{x-3}$ ( 4)

1.1.3. $0 < 6x^2 - 7x - 3$ ( 3)

1.1.4. $2.\sqrt{3-x} - 24 = 3x$ ( 5)

1.1.5. $6x^{-\frac{4}{3}} + 7x^{-\frac{2}{3}} - 24 = 0$ ( 6)

1.1.6. $4^{x-1} + 2^{2x} = 5.\sqrt[3]{2}$ (without the use of a calculator) ( 5)

1.2. Solve for $x$ and $y$ :

$y^2 - 2yx - x^2 = 31$ and $2y - x = 11$ ( 6)

1.3. Discuss the nature of the roots of :

$k^2x^2 - 4 = kx - x^2$

where $k \in \mathbb{R}$. ( 4)

1.4. Without the use of a calculator, simplify fully :

$$\frac{\sqrt{27} - 5.\sqrt{243}}{\sqrt{15}}$$

leaving your answer with a rational denominator. ( 5)

**[40]**

## QUESTION 2

2.1. Given the arithmetic series :

$$(x+1)+(-2x-8)+(3-x)+\cdots = 15\,476$$

2.1.1. Calculate the value of $x$, showing that it will be $-5$. ( 2)

2.1.2. Hence, determine the number of terms in the given arithmetic series. ( 6)

2.2. The 88th term of a quadratic sequence is $-23\,647$. The general term of the first differences is given by $-6n-8$.

2.2.1. Write down the first three first differences of the quadratic sequence. ( 1)

2.2.2. Hence, determine an expression for the general term of the quadratic sequence. ( 4)

2.3. Given : $\frac{1}{6}+\frac{5}{42}+\frac{5}{56}+\frac{5}{72}+\ldots$

For this series, determine :

2.3.1. $S_1$, $S_2$ and $S_3$ ( 1)

2.3.2. Hence, an expression for $S_n$, the sum of the first $n$-terms of the series. ( 2)

**[16]**

## QUESTION 3

3.1.1. Prove that the sum of the first $n$-terms of a geometric series whose first term is $a$ and having a common ratio of $r$, is given by :

$$S_n = \frac{a(r^n - 1)}{r - 1}$$ ( 4)

3.1.2. Evaluate : $\sum_{k=3}^{14} 10.\left(-\frac{3}{2}\right)^{2-k}$ ( 5)

3.2. Given the converging infinite geometric series :

$$\frac{1-2x}{3} + \frac{1-4x+4x^2}{9} + \frac{(1-2x)^3}{27} + \ldots$$

3.2.1. Determine an expression for $r$, the common ratio of the series, in terms of $x$. ( 1)

3.2.2. Hence, determine the values of $x$ for which the series will converge. ( 4)

3.2.3. Now, if $x = -\frac{1}{2}$ determine $S_\infty$. ( 3)

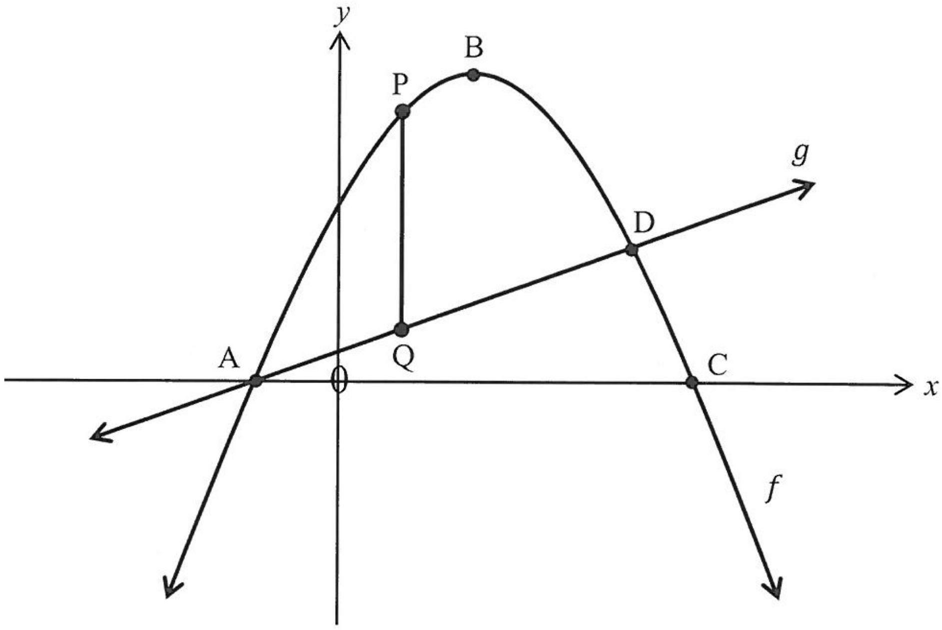
**[17]**

## QUESTION 4

4. The graphs of $f$ and $g$ have the defining equations of

$$f(x) = -2(x-1)^2 + 18 \quad \text{and} \quad g(x) = 2x + 4$$

PQ is a vertical line whose $x$ value lies between the $x$ values of points A and D.
B is the turning point of $f$.




4.1. Determine the coordinates of :

4.1.1. A and C (label your answers clearly) (3)

4.1.2. B (2)

4.1.3. D (5)

4.2. Calculate the maximum length of PQ. (4)

4.3. Solve for $x$ :

4.3.1. $\dfrac{2x+4}{-2(x-1)^2+18} \leq 0$ (2)

4.3.2. $x.f(x) \geq 0$ (2)

4.4. Calculate the average gradient of $f$ between $x = -3$ and $x = 6$. (3)

4.5. Now, consider the inverse of $f$, $y = f^{-1}(x)$.

4.5.1. State the coordinates of the turning point of $f^{-1}$. (1)

4.5.2. Will $f^{-1}$ be a function ? (1)

**[23]**

## QUESTION 5

5. Given : $f(x) = \dfrac{7 - 5x}{x - 2}$

5.1. Show that $f$ can be written as : $f(x) = -\dfrac{3}{x-2} - 5$ ( 2)

5.2. State the range of $f$. ( 1)

5.3. Sketch the graph of $f$. All *relevant* details must be shown on the graph. ( 5)

**[ 8]**

## QUESTION 6

6.1. Determine the coordinates of A', the reflection of $A(-7; 2)$ in the line $y = -x + 4$. ( 2)

6.2. The graph of $g(x) = \dfrac{a}{x+p} + q$ has the following axes of symmetry :

$y = -x - 4$ and $y = x + 3$

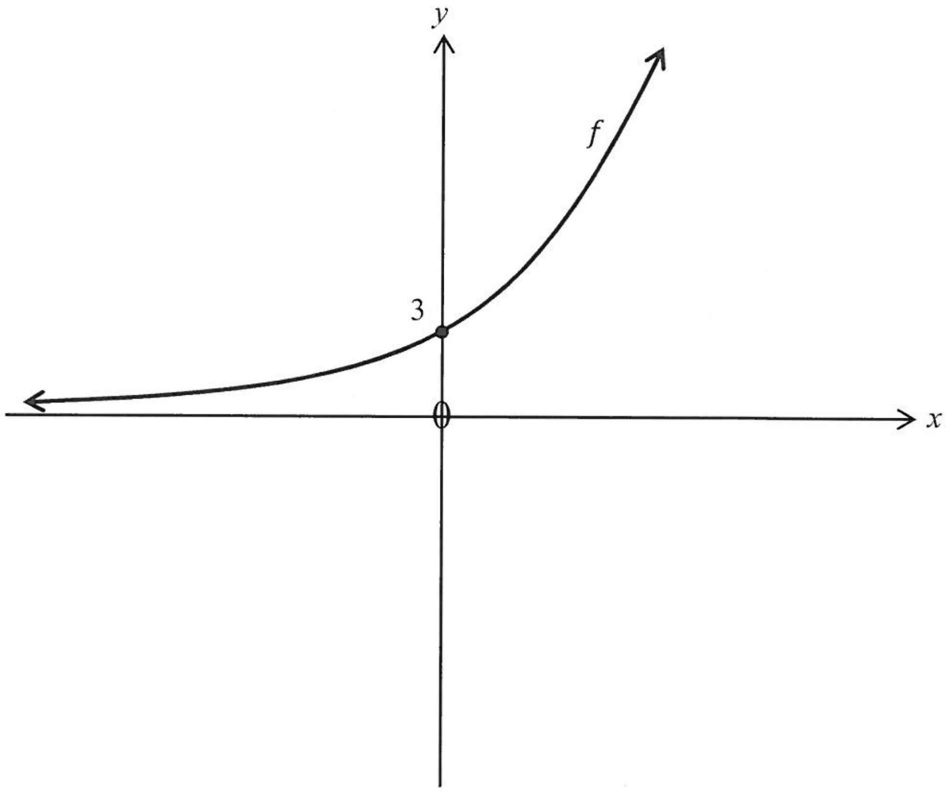
Calculate the values of $p$ and $q$. ( 3)

**[ 5]**

## QUESTION 7

**USE THE ANSWER SHEET PROVIDED**

7. The graph of $f(x) = a.2^x$ is shown below :




7.1. Determine the value of $a$. ( 1)

7.2. On the axes given on the provided answer sheet, sketch :

7.2.1. $y = x$ (accurately), and ( 2)

7.2.2. $f^{-1}$, the inverse of $f$. ( 2)

7.3. State the domain of $f^{-1}$. ( 1)

7.4. Determine the equation of $f^{-1}$ in $y$-form. ( 2)

7.5. If $f$ were translated

- 5 units vertically downwards, and
- 4 units horizontally to the left

to become $g$, state the equation of $g$ in $y$-form. ( 2)

**[10]**

## QUESTION 8

8.1. How many months will it take for an investment of R 5 000 to grow into R 6 522,25 in an account that earns compound interest of 8 % per annum compounded monthly ? ( 4)

8.2. A vehicle depreciates **by** a third of its value in 10 years, according to the diminishing balance method. Calculate the rate of depreciation. Leave your answer as a percentage. ( 4)

8.3. Convert a nominal interest rate of 7,5 % per annum compounded quarterly to an effective annual interest rate. Leave your answer as a percentage. ( 3)

**[11]**

## QUESTION 9

9.1 When $3x^3 - 7ax^2 + 4x - 5$ is divided by $x + 2$, the remainder is 8. Calculate the value of $a$. ( 2)

9.2. Given : $f(x) = 30x^3 - x^2 - 61x + 12$

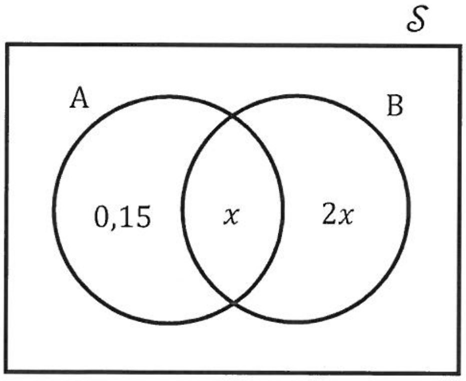
9.2.1. Use the factor theorem to show that $3x - 4$ is a factor of $f$. ( 2)

9.2.2. Hence, factorise $f$ fully. ( 3)

**[ 7]**

## QUESTION 10

10.1. For two events, A and B :




it is known that

- P(A only) = 0,15
- P(A and B) = $x$
- P(B only) = $2x$
- P( (A or B)′ ) = 0,25

10.1.1. Calculate the value of $x$, showing that it will be 0,2. ( 2)

10.1.2. Are A and B independent events ? Justify your answer showing all *relevant* working out. ( 4)

10.2. A bag contains 3 blue marbles and 8 red marbles.
A marble is drawn from the bag, and not returned to the bag.
A second marble is drawn from the bag.

10.2.1. Represent the given events in the form of a tree diagram.
All *relevant* details must be shown on the diagram. ( 4)

10.2.2. Calculate the probability that two marbles of the same colour will be drawn. ( 3)

**[13]**

| **TOTAL** | **150** |
|---|---|

## INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}$ ; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$ ; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In $\Delta ABC$: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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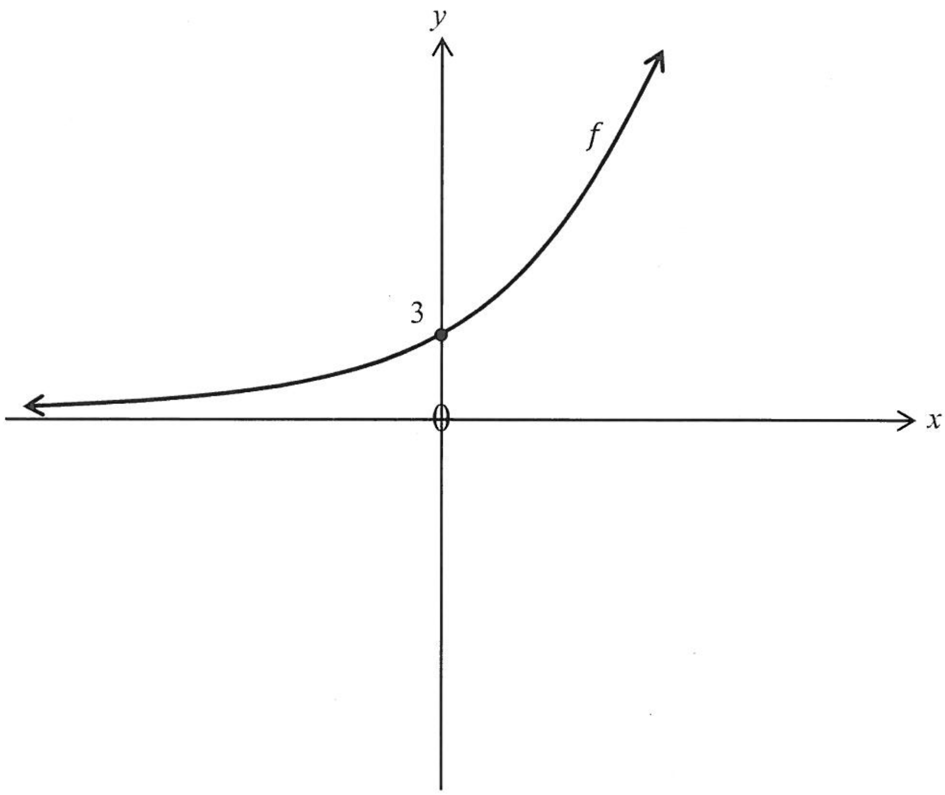
**ANSWER SHEET for QUESTION 7**

7.

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